

Fracture Functions* †

L. Trentadue ^{a b c}

^aINFN, Sezione di Roma II, and Dipartimento di Fisica, Università di Roma "Tor Vergata",
via della Ricerca Scientifica, 1, I-00133 Roma, Italy

^bDipartimento di Fisica, Università di Parma, 43100 Parma, Italy

^cpresent address:

Theory Division, CERN, Geneva, Switzerland

We present a new approach to semi-inclusive hard processes in QCD by means of *Fracture Functions*, hybrids between structure and fragmentation functions. We briefly motivate and describe it together with a list of possible applications.

1. Introduction

Asymptotic freedom [1] together with general factorization theorems [2] have made possible to predict a large variety of perturbative hard processes [3]. In the QCD improved parton model experimental cross-sections can be computed by convoluting some uncalculable, but process independent, quantities with process-dependent, but calculable, elementary cross-sections. For any given process, initiated by the hadrons A and B : $A + B \rightarrow A' + B' + \dots$, it is possible to write it in terms of a pointlike, partonic cross-section $d\sigma$ convoluted with suitably defined structure and fragmentation functions, F_A^i and $D_j^{A'}$. The mass-singularities plaguing the radiatively corrected distributions, can be absorbed in the structure and fragmentation functions. The cross-section written in the factorized form is:

$$d\sigma(Q^2) = \sum_{k, \dots} \int dx_i dx_j \dots dz_k \dots F_A^i(x_i, Q^2) F_B^j(x_j, Q^2) \dots d\sigma_{ij}^{k, \dots}(Q^2) D_k^{A'}(z_k, Q^2) \dots \quad (1)$$

where $d\sigma_{ij}^{k, \dots}(Q^2)$ is the pointlike partonic cross-section. The universal structure and frag-

mentation functions $F_A^i(x, Q^2)$ and $D_{i,A}(z, Q^2)$ do obey the evolution equations:

$$F_A^i(x, Q^2) = \sum_b \int_x^1 \frac{dz}{z} E_i^b\left(\frac{x}{z}, Q^2, Q_0^2\right) \cdot F_A^b(z, Q_0^2) \quad (2)$$

$$D_i^A(x, Q^2) = \sum_b \int_x^1 \frac{dz}{z} E_i^b\left(\frac{x}{z}, Q^2, Q_0^2\right) \cdot D_b^A(z, Q_0^2)$$

Universality means that the structure functions, which can be measured in deep inelastic lepton-hadron collisions, can be then used to compute either the same process or a completely new hard reaction at a different scale.

2. A new look to inclusive processes

Let us reformulate, in a slightly different way, some well known QCD results. Inclusive hard processes can be divided in two classes: *totally inclusive* and *semi-inclusive* ones. Furthermore, within each of these classes, one can list processes according to the number of the hadrons appearing in the initial and in the final state n_i and n_f respectively.

We may consider a process with just leptons in the initial state $n_i = 0$ $e^+e^- \rightarrow$ *hadrons*. If no particular hadron is singled out in the final

*Work done in collaboration with G. Veneziano.

†Invited talk at the "QCD 94" Conference, Montpellier (France) July 7-13th 1994.

state then $n_f = 0$. In this case the process is completely calculable:

$$\sigma(e^+e^- \rightarrow H) = \sum_x \sigma(e^+e^- \rightarrow x) \quad (3)$$

where, if H is anything, the sum over x runs over any partonic final state. If H represents three jets, x will be any number of partons in a three-jet configuration and so on. The sum over the final partons will eliminate all infrared and collinear singularities and the cross-section will have a finite perturbative expansion in α_s [4][5].

Let us consider now processes in which one hadron is present in the initial state $n_i = 1$, thus, typically, deep inelastic lepton hadron scattering [6]. The cross-section for some hard process H : $\sigma(l + N \rightarrow l' + H + X)$, in which no particular hadron is singled out in the hadronic final state $H + X$, is well known to take the form [3]:

$$\sigma_{l+N \rightarrow l'+H+X} = \sum_j \int_0^1 \frac{dx}{x} F_N^j(x, Q) \sigma_H^j(x, Q) \quad (4)$$

Where Q is the virtuality of the photon and $F_N^j(x, Q)$ is the structure function of the target N . H can represent any hard process as a jet, many jets, a photon and a jet, two heavy quarks, etc... The factorized form of eq.(4) corresponds to the fact that, as result of the hardness of the collision, the final state consists of two well separated clusters of particles, one (denoted by X) originating from the target fragmentation and from the evolution of the active parton and the other (denoted by H) coming from the subsequent hard interaction of the active parton with the lepton. In addition there will be some wee partons (or soft hadrons) which cannot be unambiguously attributed to either H or X . In the case of hadron-hadron hard collisions, the cross-sections for $A + B \rightarrow H + X_A + X_B$ can be analogously factorized as:

$$\sigma_{A+B \rightarrow H+X_A+X_B} = \sum_{ij} \int \frac{dx_i}{x_i} \frac{dx_j}{x_j} \cdot F_A^i(x_i, Q) F_B^j(x_j, Q) \sigma_{hard}^{i+j \rightarrow H}(x_i, x_j; Q) \quad (5)$$

Eq.(5) does not contain, under this factorization hypothesis, new uncalculable quantities besides the

ones we can already measure in deep inelastic scattering.

If a single hadron is detected in the final state $n_f = 1$ then the simplest case corresponds to the cross section:

$$\frac{d\sigma_{e^+e^- \rightarrow h+X}}{dz} = \sum_i \sigma_{e^+e^- \rightarrow q_i \bar{q}_i} D_i^h(z, Q) \quad (6)$$

which can be used to determine from the data the perturbatively uncalculable fragmentation function $D_i^h(z, Q)$. Thus, in this case, even processes with no initial hadron provide important non-perturbative information. The process $l + A \rightarrow l' + h + H + X$, according to our previous discussion, will receive contributions from two well separated kinematical regions for the produced hadron h :

$$\begin{aligned} \sigma_{l+A \rightarrow l'+h+H+X} &= \sigma_{current} + \sigma_{target} \\ &= \sigma_{l+A \rightarrow l'+(h+H')+X} + \sigma_{l+A \rightarrow l'+H+(h+X')} \end{aligned} \quad (7)$$

For the first term, apart from the factor arising from target structure function, no knowledge other than the one on fragmentation functions D is needed. Such "current" contribution has been widely discussed in the literature [7] and we will not examine it here. We shall instead concentrate on the second term claiming that its description does require a new non-perturbative (but measurable) quantity, a fragmentation-structure or "fracture" function [8]:

$$\sigma_{target} = \int_0^{1-z} \frac{dx}{x} M_{A,h}^i(z, x; Q) \sigma_{hard}^i(x, Q) \quad (8)$$

This form clearly implies a new factorization which will permit to describe the full target fragmentation in terms of the single function M without separating, as it is usually done, the contributions of the active parton and that of the spectators [9]; The factorized form in eq.(8) implies that, once M is measured in deep inelastic scattering no extra input is needed in order to compute analogous quantities in *hadron-hadron* collisions. Furthermore, it becomes possible to introduce in QCD new uncalculable, but measurable and universal functions, that we call "fracture" functions telling us about the *structure* function of a given target hadron once

it has *fragmented* (hence its name) into another given final state hadron. Fracture functions depend upon two hadronic and one partonic label and on two momentum fractions, a Bjorken x and a Feynman z variable:

$$M = M_{p,h}^j(x, z; Q) . \quad (9)$$

One can also say that M measures the parton distribution of the object exchanged between the target and the final hadron, without making any model about what that object actually is. As for ordinary structure functions, the importance of measuring such an object will be twofold: (1) it will teach us about the structure of hadronic systems other than the usual targets, and (2) it can be used as input for computing other hard semi-inclusive processes at other machines, such as some future hadronic collider. Furthermore, it has been recently observed in a next-to-leading evaluation of single particle cross-sections, that an entire class of collinear divergences of hadrons emitted along initial state directions are naturally absorbed by fracture functions [12].

3. Properties of Fracture functions

In order to take into account the running of α_s , it is convenient to replace (see, e.g., Ref.[5]) the evolution variable Q^2 , representing the hard scale of the process, by the variable Y defined by:

$$Y = \frac{1}{2\pi b} \ln \left[\frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)} \right] \quad (10)$$

with μ the renormalization scale, $\alpha_s(Q^2) = (b \ln \frac{Q^2}{\Lambda^2})^{-1}$ and the one loop β -function coefficient b given by $12\pi b = 11N_C - 2N_F$. The evolution equation for the fracture function $M_{p,h}^j(x, z; Q)$ feels the two distinct mechanisms of hadron production in the target fragmentation region, the one coming from the evolution of the active parton and the one due to fragmentation of the spectators. As a result, the evolution equation for $M_{p,h}^j(x, z; Q)$ has two terms [8]:

$$\begin{aligned} \frac{\partial M_{p,h}^j(x, z; Y)}{\partial Y} &= \int_{\frac{x}{1-z}}^1 \frac{du}{u} P_i^j(u) M_{p,h}^i\left(\frac{x}{u}, z; Y\right) \\ &+ \int_x^{\frac{x}{x+z}} \frac{udu}{x(1-u)} \hat{P}_i^{jl}(u) D_l^h\left(\frac{zu}{x(1-u)}, Y\right) \end{aligned} \quad (11)$$

$$\cdot F_p^i\left(\frac{x}{u}, Y\right)$$

with $P_i^j(u)$ and $\hat{P}_i^{jl}(u)$ the regularized and real [5] Altarelli-Parisi vertices, respectively.

$D_l^h(z, Y)$ represents the fragmentation function of the parton l into the hadron h and $F_p^i(x, Y)$ is the ordinary deep inelastic structure function. x and z are the Bjorken variable of the i -parton and the Feynman variable of the hadron- h . It can be seen [8] that eq.(11) has the solution:

$$\begin{aligned} M_{p,h}^j(x, z; Y) &= \int_x^{1-z} \frac{dw}{w} E_i^j\left(\frac{x}{w}, Y - y_0\right) \\ &M_{p,h}^i(w, z; y_0) + \int_{y_0}^Y dy \int_{x+z}^1 \frac{dw}{w^2} \int_{\frac{x}{w}}^{1-\frac{z}{w}} \frac{du}{u(1-u)} \\ &\cdot E_k^j\left(\frac{x}{wu}, Y - y\right) \hat{P}_i^{kl}(u) D_l^h\left(\frac{z}{w(1-u)}, y\right) F_p^i(w, y). \end{aligned} \quad (12)$$

The first term takes the hadron distribution at a given arbitrary scale y_0 and evolves it to the hard scale Y by means of the perturbative “evolution” function $E_i^j(\frac{x}{w}, Y - y_0)$ determined by the evolution equation $\frac{\partial}{\partial y} E_i^j(x, y) = \int_x^1 \frac{du}{u} P_k^j(u) E_i^k(\frac{x}{u}, y)$. The second term describes the perturbative evolution from y_0 to Y of the shower generated by the active parton. The shower generates perturbatively an inclusive distribution for the parton l which finally fragments into h . The second term in (12) contains F and D but not the fracture function M itself. It can be also shown [8] that:

- the solution given in eq. (12) does not depend on the arbitrary scale y_0 , chosen as the starting point of the evolution i.e.:

$$\frac{\partial}{\partial y_0} M_{p,h}^j(x, z; Y) = 0 . \quad (13)$$

- $M_{p,h}^j(x, z; Y)$ satisfies the natural momentum sum rule:

$$\begin{aligned} \sum_h \int dz z M_{p,h}^j(x, z; Y) &= (1 - x) \\ &\cdot F_p^j(x, Y). \end{aligned} \quad (14)$$

accounting for s-channel unitarity constraints.

4. Applications

Leaving to further work a more detailed analysis, let us list possible applications of fracture functions.

- One can simply consider $M_{N,h}^j(x, z; Q)$ for large z and define that to be the structure function of the leading trajectory that can be exchanged between the target (here a nucleon) and the observed hadron. More generally, on the basis of the Regge-Mueller analysis of inclusive cross-sections, we may expect, as $z \rightarrow 1$, an expansion of the type:

$$M_{N,h}^j(x, z; Q) \rightarrow \Sigma_R (1-z)^{1-2\alpha_R} \cdot F_R\left(\frac{x}{1-z}; Q\right) \quad (15)$$

where the sum is over different Regge poles of intercept α_R . $F_R(x; Q)$ may be defined to be the structure function of the R th Reggeon exchanged between N and h . Such a parametrization can be particularly suitable to describe diffractive processes recently observed at the Tevatron and HERA [11].

- One could compare fracture functions for various quantum numbers of the $N - h$ system, and, in particular, the relative amounts of valence quarks, sea quarks and gluons in various channels. Gluon-rich distributions for vacuum quantum numbers ($N = h = p$), i.e. the so-called Pomeron structure function [10]. At the opposite extreme, for so-called exotic quantum numbers, we should find fracture functions which are very rich in valence quarks. Examples of this type, with a proton target, are $h = \bar{p}$ and $h = \bar{K}$ in which the fracture function contains six and five quarks, respectively.
- Various deep inelastic lepton-hadron processes can be used in order to disentangle quark and gluon fracture functions for

various h . Thus, while the ordinary semi-inclusive cross-section can be used to measure the quark distribution, production of heavy quarks can give the gluon distribution. For W and Z production via the Drell-Yan process, for example, it would be convenient to trigger on final hadrons which give quark-rich fracture functions. If, instead, one is interested in Higgs production via the gluon-gluon fusion process, gluon-rich distributions will have to be preferred.

- A possible application of fracture functions is also to polarized lepton-hadron processes and to the question of the so-called spin crisis [13]. The latter simply means that the matrix element of the flavour-singlet axial current in the proton is significantly smaller than one expects in the naive quark model. Due to the $U(1)$ anomaly the spin problem and the $U(1)$ problem are indeed related [15]. The question still remains of whether the smallness of the proton spin is related really to the nature of the target (the proton here) or whether it is a more general (i.e. target independent) property of the singlet axial current [14]. Fracture functions can provide new informations by allowing one to measure matrix elements of the axial currents in the $p + h$ state.

Fracture functions could open new possibilities for studying hadron structure and for predicting hard processes. It would be interesting to see how they do compare with real data of deep inelastic or hadron hadron scattering.

Acknowledgements

I would like to thank Gabriele Veneziano for the stimulating and pleasant collaboration. I would also like to thank Stephan Narison for inviting me to this lively Conference.

This research is supported in part by the EEC Programme "Human Capital and Mobility", Network "Physics at High Energy Colliders", contract CHRX-CT93-0537 (DG 12 COMA).

REFERENCES

1. H. D. Politzer, Phys. Rep. 14 (1974) 129 and references therein.
2. D. Amati, R. Petronzio and G. Veneziano, Nucl. Phys. B140 (1978) 54; R.K. Ellis, H. Georgi, M. Mahaceck, H.D. Politzer and G.G. Ross, Phys. Lett. 78B (1978) 281; Nucl. Phys. B152 (1979) 285. J. Collins, D. Soper and G. Sterman, in "Perturbative QCD", A.H. Mueller Editor, World Scientific, 1989.
3. G. Altarelli, Phys. Rep. 81 (1982) 1.
4. T. Kinoshita, Journ. of Math. Phys., 3,650 (1962); T.D. Lee and M. Nauenberg, Phys. Rev. 133 (1964) 1549; G. Sterman and S. Weinberg, Phys. Rev. Lett. 39 (1977) 1436; E. Fahren, Phys. Rev. Lett. 39 (1977) 1587; S. Catani, G. Turnock, B. R. Webber and L. Trentadue, Phys. Lett. 263B (1991) 491.
5. K. Konishi, A. Ukawa and G. Veneziano, Phys. Lett. 78B (1978) 243; 80B (1979) 259; Nucl. Phys. B157 (1979) 45.
6. R. Taylor, An Historical Review of Lepton Proton Scattering, SLAC-PUB-5832, June 1992, presented at the 1991 SLAC Summer Institute on Particle Physics: Lepton Hadron Scattering;
7. E. Berger, Semi-Inclusive Inelastic Electron Scattering from Nuclei, Proceedings of the NPAS Workshop on Electronuclear Physics with Internal Targets, SLAC, 1987, SLAC-report 316, R. G. Arnold and R. C. Minehart Eds., p. 82.
8. L. Trentadue and G. Veneziano, Phys. Lett. B323 (1994) 201.
9. For a discussion of this and related arguments within the parton model framework see R.P. Feynman, Photon-Hadron Interactions, W.A. Benjamin Ed., 1972, lecture 55.
10. P. V. Landshoff, The Structure of the Pomeron, presented at the 27th Rencontre de Moriond: Perturbative QCD and Hadronic Interactions, March 22-28, 1992; Editions Frontieres, J. Tran Thanh Van Ed., p. 393 and references therein.
11. See the talks of V. Del Duca and G. Levman at this Conference.
12. D. Graudenz, preprint CERN-TH.7300/94, June 1994.
13. For a review, see, e.g., G. Veneziano, The spin of the proton and the OZI limit of QCD, in "From Symmetries to Strings: forty years of Rochester Conferences" (Okubofest, Ashok Das Ed., WSPC 1990) p. 86; S. D. Bass and A. W. Thomas, The spin structure of the nucleon, Adelaide preprints, ADP-92-183/T115 (1992), ADP-93-218/T136 (1993).
14. S. Narison, G.M. Shore and G. Veneziano, preprint CERN-TH.7223, April 1994.
15. G. Veneziano, Mod. Phys. Lett. A4 (1989) 1605; G.M. Shore and G. Veneziano, Phys. Lett. 244B (1990) 75, Nucl. Phys. B381 (1992) 3; G.M. Shore, talk at this Conference.